CHAPTER 8 -- ROTATIONAL MOTION

QUESTION SOLUTIONS

8.1) A ball and a hoop of equal mass and radius start side by side and proceed to roll down an incline. Which reaches the bottom first? Explain.

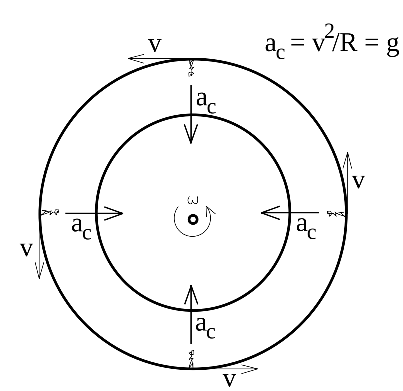
Solution: Which body has the greatest rotational inertia (i.e., moment of inertia)? It's the hoop. Why? Because on average, there is more mass farther out away from the central axis of the hoop than from the central axis of the ball. So which has the greatest angular acceleration? The body that has the least resistance to changing its rotational motion (i.e., the body with the least rotational inertia). As that is the ball, the ball will reach the bottom of the incline first.

8.2) If you drive a car with oversized tires, how will your speedometer be affected?

Solution: A speedometer really has two parts to it. Attached to the wheel is a sensor that counts the number of wheel rotations the tire executes per unit time. That information is passed on to a meter located on the dashboard (i.e., what you and I would call *the speedometer*) which converts that data into a number that corresponds to the *translational distance traveled per unit time* (i.e., the velocity) in *miles per hour*. So let's assume that your car has stock tires on it, and a sensor reading of 7 turns per second corresponds to a distance traveled of approximately 44 feet per second. This would present itself on the speedometer as a velocity of 30 miles per hour (in fact, these numbers are approximately accurate--88 ft/sec corresponds to 60 mph; that means 44 ft/sec corresponds to 30 mph; a 1 ft radius tire has a circumference of $2\pi r = 2(3.14)(1\ ft) = 6.28\ ft$, so to cover 44 ft in a second, the wheel will have to rotate *44 ft/sec)/(6.28 ft/rotation)= 7 rotations per second* approximately). What happens when oversized tires are put on your car and the wheels are made to rotate, once again, 7 turns per second? The distance now traveled in a second will be greater than 44 ft because the tires now have a larger circumference, but the sensor will still register 7 rotations per second and the speedometer will still convert and present this as 30 mph. In short, the actual speed will be greater than the speed presented by the speedometer, which means the speedometer will register a speed that is slower than the actual speed of the car.

8.3) Assume global warming is a reality. How will the earth's *moment of inertia* change as the Arctic ice caps melt?

Solution: If the Arctic ice caps melt, the released water will respond to the earth's rotation and flow outward away from the axis of rotation (i.e., toward the earth's equator). With more mass farther away from the axis of rotation, the earth's rotational inertia (i.e., moment of inertia) will increase.



8.4) Artificial gravity in space can be produced by rotation.

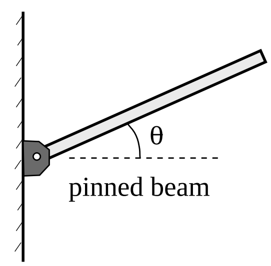
a.) How so?

Solution: If you rotate a hoop-shaped space station about its central axis, the normal force provided by the station's floor will push objects on the floor (like a person) out of straight line motion and into circular motion. A person standing on the floor will, therefore, feel an *upward force* that will superficially appear to be no different than the force he or she might experience while standing on the earth. The difference is that because the apparent force is caused by the rotation of the station, we can relate that force and its associated acceleration with the velocity of the station. That is, centripetal force requires a centripetal acceleration of $a_c = v^2/r$, where v is the translational velocity of the station's floor and r is the floor's radius). Putting that acceleration equal to g yields $g = a_c = v^2/R$, or $v = (Rg)^{1/2}$. In other words, a floor rotating about a fixed point r units away with a translational velocity of $v = (Rg)^{1/2}$ will create an apparent acceleration equal to g, and anyone standing on such a floor will not be able to tell the difference between that situation and a situation in which the floor is stationary in the earth's gravitational field. In both cases, the individual will feel as though they weigh mg. Of additional interest here is the fact that the translational velocity v of the floor will be related to the angular velocity ω of the station by $v = r\omega$, so the angular velocity required to make the acceleration g will be $\omega = v/R = (Rg)^{1/2}/R = (g/R)^{1/2}$.

b.) Assume a rotating space station produces an artificial acceleration equal to g. If the rotational speed is halved, how will that acceleration change?

Solution: We know that $v = r\omega$, $a_c = v^2/r$ and, as a consequence, $a_c = (r\omega)^2/r$. Putting $a = g$, we can write $g = (r\omega)^2/r$. From that relationship, halving ω evidently quarters the centripetal acceleration, and a_c goes to $g/4$.

8.5) Make up a conceptual graph-based question for a friend. Make it a real stinker, but give enough information so the solution *can* be had (no fair giving an impossible problem).



8.6) A beam of length L is pinned at one end. It is allowed to freefall around the pin, angularly accelerating at a rate of $\alpha = k\cos\theta$, where k is a constant. If you know the angle at which it started its freefall, is it kosher to use rotational kinematics to determine the angular position of the beam after $t = .2$ *seconds*? Explain.

Solution: Rotational kinematics are predicated on the assumption that the *angular acceleration* is a CONSTANT. This angular acceleration function changes with the angle, so it is not constant and rotational kinematics would not be applicable here.

8.7) The angular velocity of an object is found to be $-4\,\mathbf{j}$ *radians per second*.

a.) What does the unit vector tell you?

Solution: The unit vector defines the *AXIS about which the body rotates*. That axis will always be perpendicular to the plane in which the motion occurs, so the unit vector also allows you to identify the plane of rotation.

b.) What does the negative sign tell you?

Solution: The negative sign tells you that the object's rotation is clockwise within the plane of motion as viewed from the positive side of the *y axis* (you *always* view from the positive side). This can be seen using the right hand rule. Position your RIGHT hand in hitchhiker position and orient your thumb so that it is in the *negative **j** direction* (i.e., down the page along the axis of rotation). In that position, your right hand will curl in the clockwise direction (again, as viewed from above on the +*y* side of the origin). That is how you determine the clockwise/counterclockwise sense of the object's motion.

c.) What does the number tell you?

Solution: The number tells you how many *radians* the object rotates through *per unit time*.

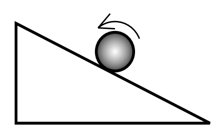
d.) How would questions *a* through *c* have changed if the *-4 **j*** had been an angular position vector?

Solution: The unit vector would define the plane in which the angle is measured (how? the unit vector would be perpendicular to that plane), the negative sign would tell you that the angular position is measured *clockwise* from the +*x axis*, and the number would tell you how many *radians* you would have to rotate through, relative to the reference axis (i.e., the +*x axis*), to get to the object's position.

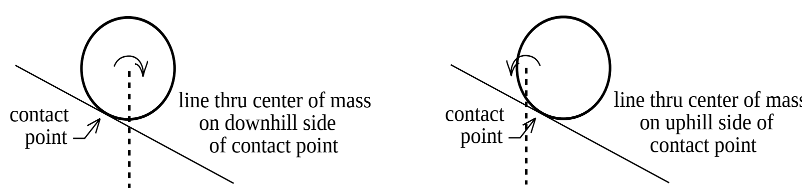
e.) How would questions *a* through *c* have changed if the *-4 **j*** had been an angular acceleration vector?

Solution: The unit vector would define the plane in which the angular acceleration takes place, the negative sign would tell you that the angular acceleration is in the clockwise direction, and the number would tell you how many *radians/second* the motion changes *per second*.

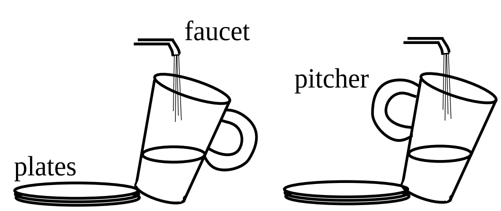
8.8) A circular disk sits on an incline. When released, it freely rolls *up* hill. What must be true of the disk?



Solution: For a normal, homogeneous disk (i.e., a disk in which the mass is uniformly distributed), the *center of mass* of the disk will be to the right of a vertical line through the contact point between the disk and our incline (see the sketch). In that case, the disk will roll down the incline. If our disk had been inhomogeneous, on the other hand, it would have been possible to position the *center of mass* to the left of that line. In that case, the disk would have rolled the other way, or *up* the hill. In other words, what's true of the disk is that it's center of mass is not at its geometric center.



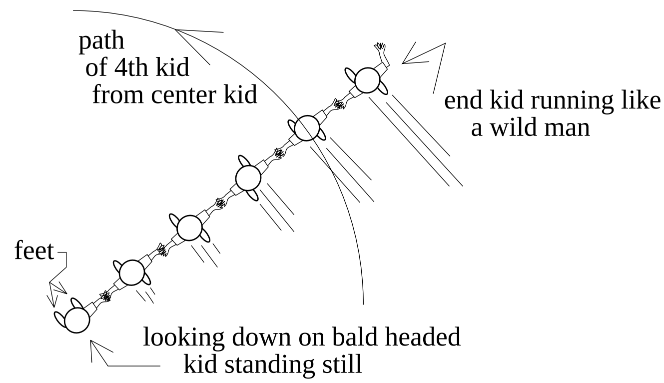
8.9) Two people want to fill up their respective water pitchers. Both use a sink in which there are



stacked plates. Neither is particularly fastidious, so each precariously perches his pitcher on the plates (notice I've made them guys?), then turns the faucet on. Which orientation is most likely to get the user into trouble? Will the trouble surface immediately or will it take time? Explain.

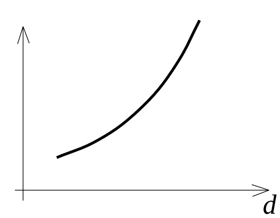
Solution: For stability, the *center of mass* of an object must be over an area of support. The problem here is that as the water fills the tipped pitchers, the *center of mass* of each pitcher/water system will creep to the right. As the pitcher on the left has more mass to the right in the first place, it is most likely to have its *center of mass* ultimately (i.e., after some time) move beyond its support and, as a consequence, tip over.

8.10) A group of kids hold hands. The kid at one end stays fixed while all the rest try to keep the line straight as they run in a circle (when I was a kid, we called this game *crack the whip*). As you can see in the sketch, the farther a kid is from the stationary center, the faster that kid has to move to keep up. If the speed of the kid one spot out from the center is v, what is the speed of the kid four spots out from the center (see sketch)? You can assume that each kid is the same size and takes up the same amount of room on the line.



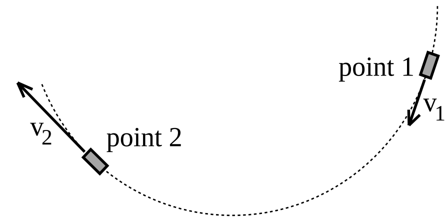
Solution: The angular velocity ω is the same for each kid (i.e., each kid is sweeping through the same number of *radians per second*), but the translational velocities get larger the farther one gets from the axis of rotation. The relationship between the angular velocity and the translational velocity of a kid r meters from the fixed axis is $v = r\omega$. That is, the translational velocity increases linearly with distance from the fixed axis. In short, a kid who is four times farther from the center than a second kid will have to travel four times as fast. For our problem, that velocity is equal to $4v$.

8.11) A light, horizontal rod is pinned at one end. One of your stranger friends places a mass 10 centimeters from the pin and, while you are out of the room, takes a mysterious measurement. She then takes the same measurement when the mass is 20 centimeters, 30 centimeters, and 40 centimeters from the end. You get back into the room to find the graph shown to the right on the chalkboard. Your friend suggests that if you can determine what she has graphed, there might be something in it for you. What do you think she has graphed?



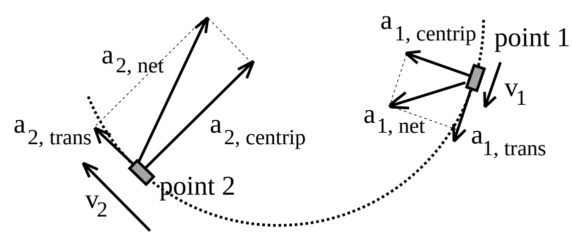
Solution: As the rotational inertia (i.e., the moment of inertia) of a point mass is proportional to the square of the distance between the mass and the axis of rotation ($I = mr^2$), the graph is most likely that of the moment of inertia of the mass about the pin.

8.12) A car rounds a corner. It goes into the curve with speed v_1 and exits the curve with greater speed v_2. Assume the magnitude of the velocity changes uniformly over the motion and the motion is circular and in the x-y plane (see sketch).



a.) On the sketch, draw the direction of acceleration of the car at the two points shown.

Solution: There are two components to the acceleration of the car. The first component forces the car into a curved path. That component will be "center seeking" and, hence, will be toward the center of the curve upon which the car happens to be traveling at a particular instance. It will also be velocity dependent as $a_c = v^2/r$. To that we must vectorially add the acceleration that increases the *magnitude* of the velocity at a particular instance (dv/dt). That acceleration will be in the direction of motion (i.e., along the line of v) and will be constant (the velocity magnitude was assumed to be changing at a constant rate). Put together, the accelerations will be as shown in the sketch. Note that although the magnitude of the translational component dv/dt will be the same throughout the motion, the centripetal component v^2/r will be greater at the second spot because $v_2 > v_1$. Also, note that the magnitude of the net acceleration changes from point to point as does the relative angle of the acceleration--something to be aware of when driving a car that is executing such a maneuver.



where $a_{centrip} = v^2/r$

and $a_{trans} = dv/dt$

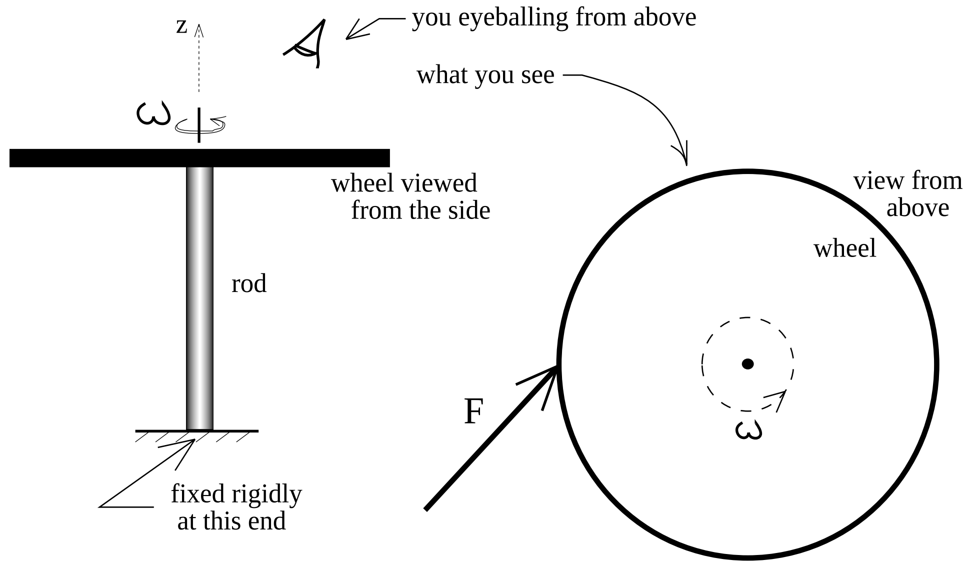
b.) Identify the car's angular acceleration at the two points.

Solution: Because the magnitude of the angular acceleration is numerically equal to a_{trans}/r, it will be constant throughout the motion. And because the direction of an angular acceleration vector is always perpendicular to the plane in which the motion occurs, its direction will also be constant (in this case, because the rotational acceleration is clockwise in the *x-y plane*, that direction will be *into the page*, or in the *-**k** direction* . . . remember, any angular parameter that is associated with counterclockwise motion will be designated as being positive while any angular parameter associated with clockwise motion will be designated as being negative).

c.) Why are angular parameters preferred over translational parameters when it comes to rotational motion?

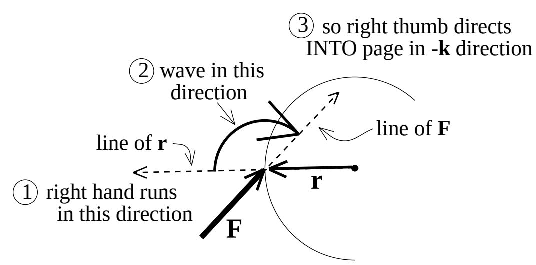
Solution: Assume you are looking at a body that is moving rotationally in the plane of this page. In comparison to the directional behavior of the body's translational parameters (i.e., its acceleration or velocity or whatever), the directional behavior of its rotational parameters is simple. That is, the direction of the vector associated with *all rotational parameters* (angular acceleration or angular velocity or whatever) will be along *one axis*--the axis perpendicular to the page. For rotational parameters, planar rotation is essentially a one-dimensional experience. What's more, if the object's translational velocity increases or decreases *uniformly* as it moves, the magnitude of a body's acceleration will change (see sketch in *Part a*) but the magnitude of the body's angular acceleration will be constant. So which would you prefer to deal with, an acceleration quantity whose magnitude and direction are constantly changing throughout the motion, or an angular acceleration quantity whose magnitude and direction are constant?

8.13) A rotating wheel is supported by a fixed rod oriented as shown. A force *F* is applied to the wheel. At the moment depicted in the sketch:



a.) In what direction is the torque due to ***F***, relative to the wheel's center?

Solution:
Formally, torque is defined as the *cross product* of the applied force ***F*** and the distance ***r*** between the point of interest (in this case, the axis of rotation) and the point at which the force acts. That is, torque $\Gamma = \boldsymbol{r}x\boldsymbol{F}$. Determining the direction of a *cross product* is usually done with the *right hand rule* (though an easier way will be presented shortly). With the *right hand rule*, the length of the right hand runs along the first vector (***r***), then the hand motions into hitch hiker position as it waves toward the second vector ***(F)***. The extended right thumb defines the direction of the *cross product*. In this case, that will be directed *into* the page in the -***k*** *direction*. THE EASY WAY TO DO THIS: Once you come to understand and believe in the *right hand rule*, note that a force that motivates a body to rotate clockwise (as viewed from the *positive* side of the axis of rotation) produces a negative torque, and a force that motivates a body to rotate counterclockwise produces a positive torque. For this problem, the force pushes the wheel clockwise, so its torque will be negative.



b.) In what direction is the wheel's resulting angular acceleration?

Solution: In the world of rotational motion, net torque and angular acceleration are the counterparts to net force and acceleration. According to Newton, the net force acting on a body is proportional to the body's acceleration (note that that means their directions are the same). Running a parallel for the rotational world, the net torque acting on a body is proportional to the body's *angular* acceleration (that means *their* directions are the same). As there are no other torques acting on the system except that of ***F***, we can conclude that the direction of the angular acceleration will be the same as the direction of the torque produced by ***F*** (i.e., in the *-**k** direction*).

c.) In what direction is the wheel's angular momentum?

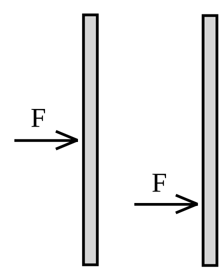
Solution: The angular momentum vector is equal to $\mathbf{L} = I\omega$. As such, the direction of the angular momentum vector will always be the same as the direction of the angular velocity vector ω. The easiest way to determine the unit vector attached to ω is to remember that the unit vector will always be perpendicular to the plane in which the rotation occurs (for our situation, that direction will be along the *z-axis*). Further, ω will be positive if the rotation is counterclockwise (as viewed from the positive side of the axis) and negative if the rotation is clockwise. In short, looking from the side of our disk, the angular velocity, hence, angular momentum vector, will be straight up in the *+**k** direction.*

Note: Although this next question has a seemingly perverse result, its weirdness will be addressed at the end of the chapter. For now, view it as an exercise in the use of the *right hand rule* coupled with a bit of thinking about the rotational version of both *Newton's Laws* and *momentum*.

8.14) Can an object that is not translating have kinetic energy?

Solution: The blade of a table saw does not translate, but it can definitely do damage to you. Rotational kinetic energy is the energy associated with rotational motion.

8.15) A meterstick sitting on a frictionless surface has a force F applied at its *center of mass*. The same force is then applied to an identical meterstick halfway between its *center of mass* and its end (see sketch).



a.) In the second situation, why might the phrase "the stick's acceleration due to the force F" be somewhat misleading?

Solution: Examining the first stick, F is applied at the stick's *center of mass*, there is no rotation, and the acceleration $a = F/m$ will be applicable to every point on the stick. But because the force on the second stick is not applied at the *center of mass*, that stick will not only accelerate translationally, it will additionally *rotate about its center of mass*. That means each point on the stick will have a different translational acceleration. There is only one part of that stick that accelerates at F/m--the stick's *center of mass*--so the phrase should have been "the acceleration of the stick's *center of mass* due to F."

b.) In the second situation, the phrase *the stick's acceleration due to F* is misleading whereas the phrase *the stick's angular acceleration due to F* is NOT misleading. How so?

Solution: This is most easily seen by examining the idea of *angular velocity*, then extrapolating to the idea of *angular acceleration*. The angular velocity of an object about its *center of mass* will be the same as the angular velocity about *ANY POINT ON THE OBJECT*. That is, if you sit at the *center of mass* and count the number of radians the object sweeps through as it rotates about you in a given time interval, the number you'll come up with will be the same as the number you'll come up with if you do the same process while sitting at any other point on the object. (That's why angular parameters are so nice.) The same can be said about an object's *angular acceleration*. If you know the angular acceleration relative to one point on the body (say, relative to the *center of mass*), you know the angular acceleration relative to *any* point on the body.

c.) Will the acceleration of each stick's *center of mass* be different in the two situations? If so, how so?

Solution: Just because there is rotation in one case and no rotation in the other doesn't mean the translational version of Newton's Second Law (i.e., $F_{net,x} = ma_x$) is no longer valid. The translational acceleration of either stick's *center of mass* will still equal the total force F acting on the stick in the *x direction* divided by the stick's mass, and that number will be the same in both cases.

d.) Will the stick's angular acceleration about its *center of mass* be different in the two situations? If so, how so?

Solution: There is no angular acceleration for the first stick because there is no torque being applied to that stick about its *center of mass* (a force acting through a point will not produce a torque about that point). As there is a torque on the second stick, the angular accelerations will be "different."

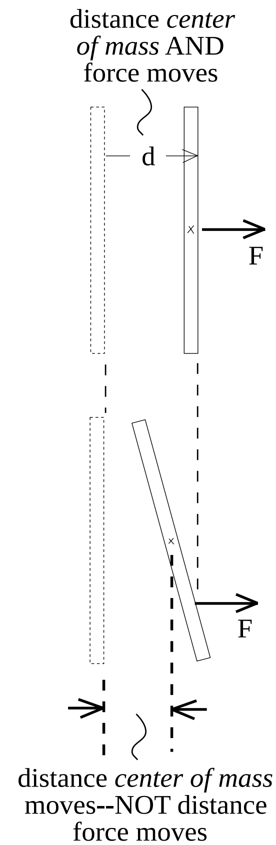
e.) Will the velocity of each stick's *center of mass* be different? If so, how so?

Solution: Because the acceleration of each stick's *center of mass* will be the same, the velocity change will be the same and the two objects will parallel one another as far as *velocity in the x direction* goes. (Note: Whenever you see the word *velocity* alone, the word refers to *translational velocity*. If you want to designate *angular* velocity, you have to do just that by using the word *angular* or *rotational* in front of the velocity term.)

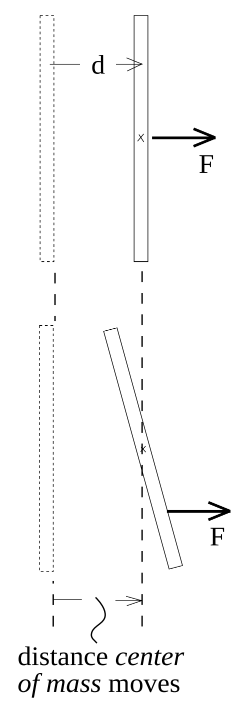
f.) Will the angular velocity about the stick's *center of mass* be different in the two situations? If so, how so?

Solution: Because there is a torque in the second case and no torque in the first case, the second case will have an angular acceleration, hence angular velocity, associated with its motion whereas the first won't.

g.) Assume the force in both cases acts over a small displacement d. How does the work done in each case compare?



Solution: This is tricky. By the definition of work, if the force acts over the same distance d in both cases, the same amount of work is done on both systems. On the other hand, as the first case is solely translational while the second case is both translational *and* rotational, it would seem more energy would be required to support the second case. To add to the confusion, we also have to contend with the fact that the *center of mass* accelerations must be the same in both cases (remember, N.S.L. doesn't cease to exist just because there's rotation in the system--a net force on a body in a particular direction, *no matter where the force is applied to the body in that direction*, is going to accelerate the body's *center of mass* by an amount equal to F/m), so it looks like the same amount of energy is being pumped into both systems when clearly we need more energy in one system than the other. So what's going on? The key lies in a subtlety of geometry. Because the force is applied at the *center of mass* in the first case, the distance the force is applied and the distance the *center of mass* actually moves are the same (see sketch). Also, it will take, maybe, *t seconds* to execute this motion. Because the force is applied at a point *other than the center of mass* in the second case, the distance the force is applied and the distance the *center of mass* actually moves are NOT THE SAME (see second sketch). In fact, the *center of mass* will travel *less distance* in that second case. Additionally, as the *center of mass* accelerations are the same in both cases, the *time* of acceleration in that second case will be *less than* the time of acceleration in the first case (this makes sense if you think about it--the force in the first case is fighting the entire inertia of the mass as it motivates that entire mass forward; the force in the second case is motivating some mass forward while some of the mass rotates backwards--the net effect is that the point at which the force is applied in the second case will shoot forward and reach d quicker, so the time of motion will be less). Bottom line: The amount of work done in both cases will be the same, but because the times of acceleration are different, the *center of mass* velocities (hence, the translational kinetic energies) will differ. That energy discrepancy is accounted for in the second case as rotational kinetic energy.



h.) Assume the force in both cases acts over a small *center of mass* displacement d (say, 2 centimeters). How does the work done in each case compare?

Solution: This is reminiscent of *Part g*. If the *center of mass* velocities of the two objects are going to parallel one another, the work needed to change each stick's translational kinetic energy must be the same. But if work is also needed to change *rotational velocity*, which happens to be the case in the second situation, then still more energy must be pumped into that system. In other words, the force F must do more work on the second stick than it does on the first stick. This may seem strange, given the fact that the *center of mass*es move the same distance, but it isn't. Think about it. Assume the force on the second stick is applied at *point*

P. As that stick rotates, does the stick's *center of mass* or *point P* move farther? *Point P* physically moves farther than the *center of mass* does. That means the force acting at that point acts *over a larger distance* than was the case with the first meterstick. It is the work done by the force acting over that extra distance that powers the rotation.

8.16) Why does a homogeneous ball released from rest roll downhill? That is, what is going on that motivates it to roll? (Hint: No, it's not just that there is a force acting! There are all sorts of situations in which forces act and rolling does not occur.)

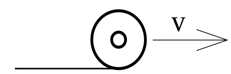
Solution: What motivates objects to roll is *torque*. In the case of the ball, you can either look at the torque relative to the *center of mass* (in that case, the torque will be produced by the friction between the ball and the incline) or the torque relative to the point of contact with the incline (in that case, the torque will be produced by gravity acting at the body's *center of mass*). In all cases, rolling doesn't happen unless there is a net torque acting on the object.

8.17) A spinning ice skater with his arms stretched outward has kinetic energy, angular velocity, and angular momentum. If the skater pulls his arms in, which of those quantities will be conserved? For the quantities that aren't conserved, how will they change (i.e., go up, go down, what?)? Explain. (Hint: I would suggest you begin by thinking about the *angular momentum*.)

Solution: As the guy pulls his arms in, there is no torque about his axis of rotation (the muscular force he applies to himself is along a line through his axis of rotation, so it produces no torque about that axis). As a consequence, ANGULAR MOMENTUM must be CONSERVED. In this case, though, the *moment of inertia* decreases as he pulls his arms in (his overall mass is getting closer to his axis of rotation diminishing his rotational inertia). As the constant angular momentum L is a function of moment of inertia I and angular velocity ω, a decrease in *moment of inertia* means an INCREASE in ANGULAR VELOCITY. In other words, as anyone who has ever watched an ice skating exhibition knows, when the arms come in, the rotation speeds up. In short, while L remains the same, I decreases and ω increases proportionally. As for kinetic energy, that is a function of angular velocity *squared* (i.e., $KE_{rot} = .5I\omega^2$). So although I goes down, ω goes up proportionally *and is squared*. The net effect is that his ROTATIONAL KINETIC ENERGY will INCREASE. (Where does the energy come from? His muscles provide it by burning chemical energy inside his body.)

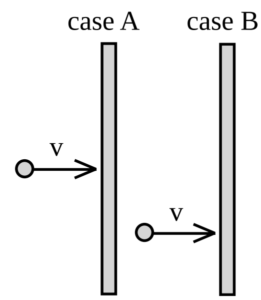
8.18) An object rotates with some angular velocity. The angular velocity is halved. By how much does the rotational kinetic energy change?

Solution: Rotational kinetic energy is equal to $KE_{rot} = .5I\omega^2$, so halving ω drops the kinetic energy by a factor of 4.



8.19) If you give a roll of relatively firm toilet paper an initial push on a flat, horizontal, hardwood floor, it may not slow down and come to a rest as expected but, instead, pick up speed. How so?

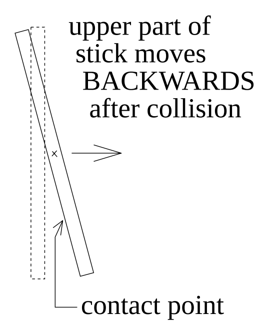
Solution: This is a fun one. As the roll lays down more and more t.p., its *center of mass* lowers. As the *center of mass* drops, gravitational potential energy is converted into kinetic energy and the body continues to roll on.



8.20) A meterstick of mass m sits on a frictionless surface. A hockey puck of mass $2m$ strikes the meterstick perpendicularly at the stick's *center of mass* (call this *case A*). A second puck strikes an identical meterstick in the same way on an identical frictionless surface, but does so halfway between the stick's *center of mass* and its end (call this *case B*).

a.) Is the average force of contact going to be different in the two cases? If so, how so?

Solution: The temptation is to think that because the collision velocities are the same, the average contact forces will be the same. It turns out that that isn't the case. First, some observations: To begin with, because the mass of the puck is *double* the mass of the meterstick, it is safe to assume that when the collision occurs, the puck will not rebound but will, instead, continue moving in its original direction with diminished speed. This will be true in both cases. It is also safe to assume that the contact point on the meterstick will leave the puck with a velocity that is greater than that of the puck (i.e., the two will separate . . . in a way, that is the only way it *can* be). It should also be noted that the force of contact will not be constant over time (that is the reason the question alludes to *average force*). At first brush, the force will be slight, growing as the impact deepens (remember, a collision between two solid objects typically occurs over a period of, maybe, several hundredths of a second). The key to untangling this question is to note that if the meterstick is easily motivated to its separation speed, the force will not have the time required to grow to the same extent that it would if the meterstick had been more inert (i.e., more difficult to motivate to separation speed). So what's happening in each case? In *case B*, the puck hits the meterstick away from the stick's *center of mass*. That means the stick both rotates and translates. As for translation, part of the stick will accelerate out away from the puck while the upper section of the stick will lag behind having rotated around the *center of mass* (see the sketch). As such, the effective inertia of the stick will be less than if the entire stick was required to accelerate uniformly as would be the situation in *case A*. As for the rotation, it will be relatively easy to motivate the meterstick into rotational motion (the moment of inertia for a rod or stick is $(1/12)mL^2$, where $L = 1$ *meter* for a meterstick and, hence, $I = m/12$--this will be small in comparison to the meterstick's translational inertia m). In short, the net inertia (rotational and translational) that must be overcome to make the meterstick separate from the puck will be relatively small in *case B*. In *case A*, on the other hand, the puck hits the meterstick dead center. There is no rotation, but the force of the collision has to overcome the entire inertia of the meterstick in motivating it away from the puck. Why? Because the meterstick will



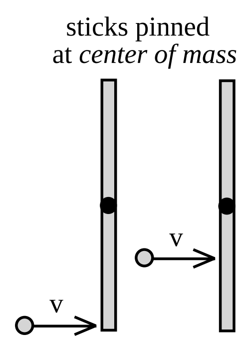
have to uniformly accelerate along its entire length. In other words, the stick will put up more resistance to changing its motion than would be the situation in *case B* and, as a consequence, will absorb more force before separation occurs. In fact, the farther out from the stick's *center of mass*, the less the average contact force will be.

b.) Is the puck's after-collision velocity going to be different in the two cases? If so, how so?

Solution: There are two ways to do this. Both make use of the fact that in *Part a* above, we concluded that the *average contact force* on the puck AND the *time of contact* was smaller in *case B* than in *case A*. With the average contact force on the puck being less in *case B*, the average acceleration will also be less in *case B*. And if that acceleration occurs over a smaller time, then the net change of puck velocity will be less and, hence, the final puck velocity will be greater in *case B* than in *case A*. The other option is to note that with the average contact force and time of contact both being smaller in *case B*, the impulse applied to the puck in *case B* will be smaller and, hence, so will its momentum change. With both systems having the same initial momentum (i.e., 2mv), that means *case B's* final velocity will be closer to its initial velocity than will be the situation in *case A*, and its final velocity will be larger.

c.) Is the puck's after-collision angular velocity (relative to the stick's *center of mass*) going to be different in the two cases? If so, how so?

Solution: You don't tend to think of a translating puck as having angular velocity about some point, but it will as long as the line of its motion doesn't pass through that point. In this case, the angular velocity of the puck about the stick's *center of mass* in *case A* will be zero because the line of that puck's motion *will* pass through the stick's *center of mass*. In the *case B*, though, the puck's angular velocity about the stick's *center of mass* will be $\omega = v/(.25 \text{ meters})$. So what is the *after-collision* angular velocity dependent upon? The torque applied to the puck about the stick's *center of mass*. The torque in the first situation will be zero (again, you are dealing with a force that acts through the stick's *center of mass*) whereas the torque on the second puck will be non-zero. With the two torques being different, the angular accelerations about each *center of mass* will differ and, as a consequence, the final angular velocities about each *center of mass* will be different.



8.21) A meterstick of mass m is pinned at its *center of mass* on a frictionless surface. A puck whose mass is $10m$ strikes and sticks to the meterstick at the .33 meter mark (i.e., .17 meters from the pin). Call this *case A*. A second meterstick experiences exactly the same situation except that its puck strikes and sticks at its end. Call this *case B*.

a.) Is energy conserved through either collision?

Solution: Energy is practically never conserved during a collision (it's close, maybe, when you're talking about charged subatomic particles interacting with one another, but in the real world, at least some energy in a collision is given up as heat or sound or in the rearrangement of material we call *deformation*). As such, energy would not be conserved in this collision (note: to add a little extra twist, some physics problems maintain that a collision is *elastic* . . . meaning energy is supposed to be conserved--this is always a contrived situation).

b.) In which case will the final angular speed be larger, and by how much?
Solution: This isn't an intuitively obvious question (well, the question may be but the answer isn't). At first glance, I'd say the greater angular velocity would belong to *case B*. It turns out that that isn't right. To see this, think about the puck for a second. Being a point mass, it's moment of inertia is $I = mr^2$. The fact that I is proportional to r^2 means that the farther out you go, the greater the puck's resistance to changing its angular motion. The torque applied to the puck when it hits a distance r meters from the pin will be rF (this is the magnitude of $\mathbf{r}\times\mathbf{F}$ when $\mathbf{r}$ and $\mathbf{F}$ are perpendicular to one another). The rotational version of Newton's Second Law states that $\Gamma = I\alpha$, or $rF = (mr^2)\alpha$, so evidently the relationship between r and α is $\alpha = (F/m)(1/r)$. With the angular acceleration and r being inversely related, the farther a given force is applied from the pin, the smaller the angular acceleration. So what does all this mean? It means we can expect that the angular acceleration will be *less* the farther out the hit occurs and, as a consequence, the angular velocity will be less the farther out the hit occurs. Does this make sense? It does *if the forces are the same*. Are they? Not if you believe the arguments that were made in the preceding problem--*Problem 9a*. So how do we proceed from here? Enter the *conservation of angular momentum* (the crowd gasps). We will do the analysis in pieces by closely examining *case B*. Observe: 1.) The meterstick begins with no angular momentum (it isn't initially moving at all). 2.) The puck has initial angular momentum, relative to the *center of mass* (remember, an object moving in a straight line has angular momentum unless its line of travel passes through the reference point). 3.) The torque on the puck due to the collision is internal to the system (that is, it's due to the puck's interaction with the meterstick), and the torque on the meterstick due to the collision is also internal to the system. 4.) The pin force provides no torque to the system about the pin, so all of the torques acting in the system are internal. 5.) *Angular momentum* is conserved when all the torques on a system are internal. 6.) The system's initial angular momentum is all wrapped up in the angular momentum of the puck. That quantity is mvd, where d is the distance between the collision point and the meterstick's *center of mass*. 6.) Noting that ω is the system's final angular velocity, the system's final angular momentum is the sum of $I_{puck}\omega$ (equal to $(m_{puck}d^2)\omega = (10md^2)\omega$) and $I_{meterstick}\omega$ (equal to $[(1/12)m_{meterstick}L_{meterstick}{}^2]\omega$). With $L = 1$ *meter* for the meterstick's length, and noting that $1/12 = .0833$, this last term becomes $.0833m\omega$. Putting it all together by equating the initial and final angular momenta yields a relationship between ω and the impact distance d in terms of the initial puck velocity v. Specifically, the relationship becomes $mvd = 10md^2\omega + .0833m\omega$. Solving yields $\omega = [10d/(10d^2 + .0833)]v$. Substituting $d = .17$ *meters* yields an angular velocity of 4.57v. Substituting $d = .5$ *meters* yields an angular velocity of 1.94v. Great jumping huzzahs! The hit farther out produces the smaller angular velocity, just as predicted with all the hand waving.

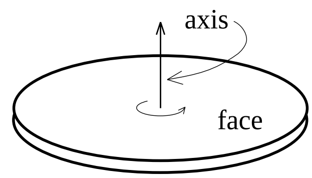
8.22) A rotating ice skater has 100 joules of rotational kinetic energy. The skater increases her *moment of inertia* by a factor of 2 (i.e., she extends her hugely muscular arms outward). How will her rotational speed change?

Solution: As there are no external torques acting, angular momentum must be conserved. That means the product of the moment of inertia I and angular velocity ω will never change. Be that the case, if I increases by a factor of 2, ω must decrease by a factor of 2.

8.23) It is easier to balance on a moving bicycle than on a stationary one. Why?

Solution: If your weight is off-center while sitting on a stationary bicycle, the bike will rotate about the ground and come crashing down. If you do the same thing on a moving bicycle, it will take a considerably greater off-set to make the bike go down. Why? A rotating wheel has angular momentum directed along its axle. Changing the direction of that angular momentum vector--something that would have to happen if the bike were to fall over--requires a sizable torque. But unless you tilt the bike considerably, no such torque is available. In other words, when you off-set your weight just a bit, the wheel's angular momentum fights the change of axis orientation allowing you time to re-set your weight appropriately.

8.24) A disk lying face-up spins without translation on a frictionless surface. At its *center of mass*, its angular velocity about an axis perpendicular to its face is measured and found to equal N. Its angular momentum at that point is measured to be M.

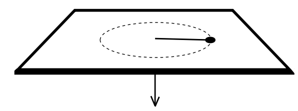


a.) Is there any other *point P* on the disk where the angular velocity about P is equal to N? Explain.

Solution: Assume you are sitting in a chair just above the *center of mass* of the disk (if you'd like, think of the disk as a huge *merry go round*). Assume also that the chair's position is oriented in a fixed direction (to do this on the *merry go round*, you would need a chair that was fixed so that it didn't rotate with the *merry go round*). The disk's angular velocity in this case tells you the number of *radians* you will observe the disk sweep through *per unit time* as it moves around underneath you. In most cases, angular velocity quantities are quoted relative to the axis of rotation, which is usually through the mass's *center of mass*. But if you were hovering above some point other than the *center of mass* (you'd have to be moving with the disk to do this, but assume you could), sitting in your fixed-direction chair, how many radians of the disk would sweep under you per unit time. The answer is *the same number of radians per second as you would have observed while sitting over the center of mass*. This makes perfect sense if you notice the following: no matter where you are located on the disk, it will always take the same amount of time for the entire disk to rotate around underneath you once (i.e., through 2π radians). As such, the angular velocity *relative to any point on the disk* will be the same as the angular velocity about any *other* point on the disk.

b.) Is there any other *point P* on the disk where the angular momentum about P is equal to M? Explain.

Solution: You might think that because the angular velocity about any point on a rotating object is the same as about any other point, the angular momentum--an angular velocity related quantity--would also be the same. The problem is that angular momentum is not only associated with angular velocity, it is also associated with *moment of inertia*. As the *moment of inertia* is going to increase as one gets farther away from the *center of mass*, the angular momentum is also going to increase. By how much? The *moment of inertia* is generally a function of r^2, so you would expect the angular momentum to increase as the *square of the distance* between the point and the disk's center.



8.25) A string threaded through a hole in a frictionless table is attached to a puck. The puck is set in motion so that it circles around the hole. The string is pulled, decreasing the puck's radius of motion. When this happens, the puck's angular velocity increases. Explain this using the idea of:

a.) Angular momentum.

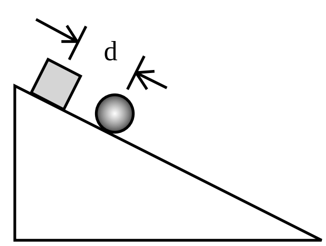
Solution: Because there are no external torques acting on the puck about the hole (in fact, there are no torques acting at all as the tension is along the line of $\boldsymbol{r}$), angular momentum will be conserved. As the radius of the circling puck diminishes, the puck's *moment of inertia* $I = mr^2$ also diminishes. For angular momentum (i.e., $I\omega$) to remain constant, therefore, ω must increase.

b.) Energy.

Solution: By pulling the string down through the hole, you are doing work ($\boldsymbol{F}\cdot\boldsymbol{d}$) on the puck. The energy must show itself somehow. It does so with an increase in the puck's rotational kinetic energy ($.5I\omega^2$). With the moment of inertia diminishing (r is getting smaller), this energy increase shows itself as an increase in the angular velocity of the puck.

8.26) When a star supernovas, it blows its outer cover outward and its core inward. For moderately large stars (several solar masses), the implosion can produce a structure that is so dense that one solar mass's worth of material would fit into a sphere of radius *less than 10 miles*. All stars rotate, so what would you expect the rotational speed of the core of a typical star to do when and if the star supernovaed? Explain using appropriate conservation principles.

Solution: As there are no external torques being applied during a supernova, the star's angular momentum will be conserved. If its mass is compressed into a very small ball, its *moment of inertia* will drop precipitously. To keep the angular momentum constant, its angular velocity must go up as much as the *moment of inertia* goes down. Stars that do this, called *neutron stars* or *pulsars*, typically have rotational speed upwards of 60 revolutions per second (think about it--an object that is 10 miles across rotating 60 times every second . . .). Pretty amazing.



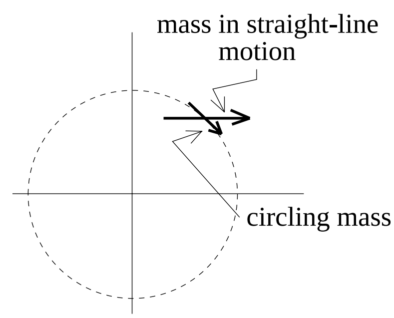
8.27) A cube and a ball of equal mass and approximately equal size are d units apart on a very slightly frictional incline plane (frictional enough for the ball to grab traction but not frictional enough to take discernible amounts of energy out of the system). By the time the ball gets to the bottom of the ramp, will the distance d be larger, smaller, or the same as it was at the beginning of the run? Use *conservation principles* to explain.

Solution: This is most easily seen by looking at the system from the perspective of energy. If both bodies drop a distance h in the same amount of time (i.e., if d remains the same throughout), the total kinetic energy should be the same for both at the end of the drop (remember, friction isn't removing a discernible amount of energy from the system). Unfortunately, there are two ways kinetic energy can show itself, as translational kinetic energy and as rotational kinetic energy. The *block's* kinetic energy is all translational. The *ball's* kinetic energy is part translational, part rotational. In other words, the block is going to pick up more translational kinetic energy than will the ball and, hence, will pick up more translational velocity than will the ball. In short, d should diminish with time.

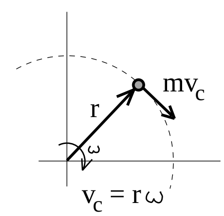
8.28) Assume global warming is a reality. How will the period of the earth's rotation change as the Arctic ice caps melt?

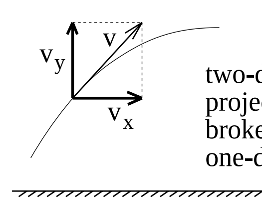
Solution: In the last chapter, we concluded that if the Arctic ice cap melts, the released water would flow outward away from the axis of rotation (i.e., toward the earth's equator) and the earth's moment of inertia I about its axis of rotation would increase. As angular momentum ($\boldsymbol{L} = I\omega$) would be conserved in this operation (there are no external torques acting on the system), an increase of the earth's moment of inertia would elicit a decrease in the earth's angular velocity.

8.29) A point mass m moving along a circular path of radius r passes a second point mass m moving in the x *direction* (see sketch). Is it possible for the two objects to have the same angular momentum and, if so, what conditions must be met for this to happen?

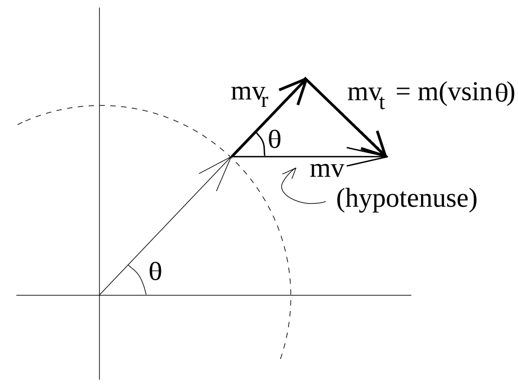


Solution: Let's start with the easy stuff. The mass that is circling about the origin clearly has an angular momentum associated with its motion. There are two ways to determine the magnitude of that angular momentum. The first is to deal solely with rotational parameters. In that case, $L = I\omega$. Remembering that the moment of inertia for a point mass is mr^2, and noting that the angular velocity ω is related to the magnitude of the instantaneous velocity by $\omega = v_c/r$ (I'm obviously defining the magnitude of the circling mass's velocity to be v_c), we can write $L = I\omega = (mr^2)(v_c/r) = mrv_c$. As a quick review, the second way to do this is to determine the magnitude of the cross product of $\boldsymbol{r}x\boldsymbol{p}$. Noting that the *line of* $\boldsymbol{r}$ and the *line of the momentum* $m\boldsymbol{v}_c$ are perpendicular, that operation yields $L = r(mv_c)\sin 90^o = mrv_c$ . . . and isn't that nice. We get the same result both ways. Now for the fun part. Because people don't intuitively associate angular properties with bodies moving in straight-line motion, one of the more obscure ideas students run into in dealing with the world of angular motion is the idea that a body moving in a straight



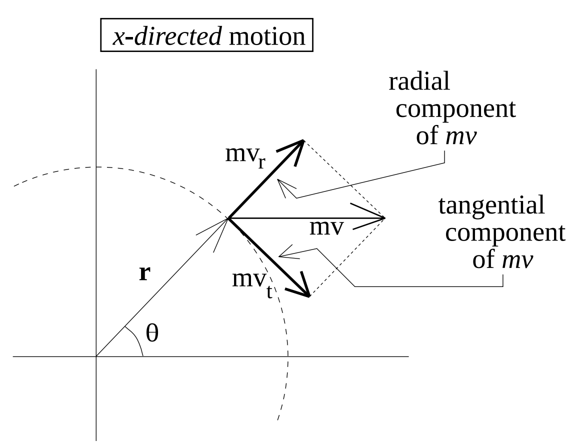


two-dimensional projectile motion broken into two independent, one-dimensional pieces line can have *angular momentum*. The reason I've included this problem is because there is a sane, conceptually appealing reason for concluding that this must be so. To see it, all you need is the right perspective. That's what I'm about to give you. But first, a small but important digression. Remember back when we talked about projectile motion. In those discussions, it was pointed out that two dimensional motion is really nothing more than *x-type* motion and *y-type* motion happening independently to the same body at the same time (see sketch). So when you attacked a projectile problem, how did you proceed? You wrote out an equation that had to do with the *x motion* (remember, with no friction, $a_x = 0$) completely ignoring what was happening in the *y direction* where the acceleration was $a_y = -9.8\ m/s^2$. You could do this kind of selective thinking because the two directions were independent of one another.

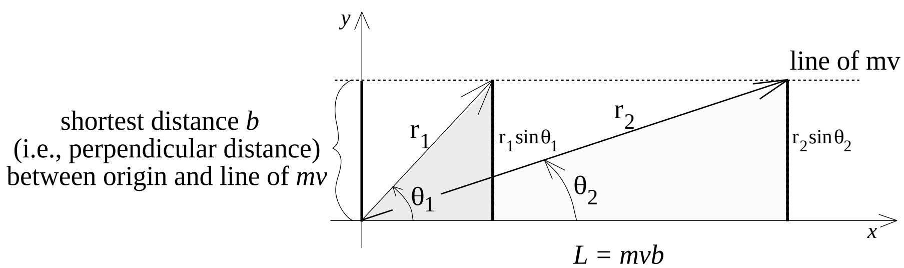


Well, we are about to do a similar thing here. Instead of thinking of the *x-directed motion* as straight-line motion (you might want to look at the sketch now), I want you to think of it as a combination of two independent bits of motion--*radial motion* (i.e., motion along a radial vector that extends from the origin to the mass) and *tangential motion* (i.e., motion that is tangent to a circle upon which the mass resides at a particular point in time). This rather strange combination is shown in the sketch (note that the tangential component of the mass's momentum mv_t and the radial component of the mass's momentum mv_r vectorially add together to equal the mass's total momentum mv). As bizarre as it may seem (to reiterate), what the sketch is suggesting is that you can think about straight-line motion as a combination of radial and tangential components of motion that are happening at the same time (just as projectile motion can be thought of *x-type* and *y-type* motion independently happening at the same time). Once you've bent your mind around this idea, it's not too big a jump to see that there is really no difference, at least instantaneously, between the tangential component of the *x-directed* momentum (i.e., mv_t) and the momentum of the circling mass mv_c. Their magnitudes may or may not be the same, but their directions are, at least instantaneously, parallel to one another.

If you are willing to accept this proposition--that *x-directed* straight-line motion can be viewed as having a *circular (i.e., angular) component* to it--then it is not at all difficult to see how that same motion might have angular momentum attributed to it. And in fact, that is exactly the case. The *x-directed* motion has an angular momentum whose magnitude is equal to the magnitude of $\mathbf{r}\mathbf{x}\mathbf{p} = \mathbf{r}\mathbf{x}(m\mathbf{v})$. Noting that the component of $m\mathbf{v}$ perpendicular to $\mathbf{r}$ is $mv_t = mv\sin\theta$ (see sketch), the evaluation of the cross product using the perpendicular component approach becomes $L = rp_t = r(mv_t) = r(mv\sin\theta) = rm(v\sin\theta)$. (Note that this was EASY to compute mathematically--you could have mindlessly, mechanically done it whether you understood the concepts being discussed here or not--the reason you've been slogging through all of this hand waving is because mindless calculations are just that, mindless--the trick is to un-



derstand why the moves are legitimate). When we compare that (i.e., $rm(v \sin \theta)$) to the circling mass's angular momentum (i.e., mrv_c), it becomes obvious that when $v_c = v_t = v \sin \theta$, the two angular momenta will be the same. Oh, and there is one more interesting bit of whimsy that should be noted. No matter what $\mathbf{r}$ happens to be at a given instant, $r \sin \theta$ will ALWAYS equal the distance labeled as b in the sketch (note that b is the perpendicular distance--read this *shortest distance*--between the point about which you are taking the angular momentum--in this case, that would be the origin of our coordinate axis--and the line of $m\mathbf{v}$). Put a little differently, no matter what r and θ are, it will always be true that $r \sin \theta = b$ (i.e., that perpendicular distance) and $L_{puck} = r(mv \sin \theta) = mv(r \sin \theta) = mvb$. Bottom line: The magnitude of the angular momentum of any object moving in straight line motion will equal the momentum mv of the object times the shortest distance b between the *line of the momentum vector* and the point about which the momentum is being taken.



PROBLEM SOLUTIONS

8.30) This problem has been included to give you a look at the basic kinematic and rotation/translational relationships. Don't spend a lot of time on it; it's more to get you familiar with the ideas than anything else.

a.) Using dimensional analysis, going from *km/hr* to *m/s* requires converting kilometers to meters and hours to seconds. The best way to do this is by simply placing the units first, then by plugging in the numbers. That is, the relationship has the units of *hours* in the denominator. That means you want to multiply by a quantity that has *hours* in the numerator and the desired *seconds* in the denominator, or *hours/second*. In that way the *hour* terms cancel out and we are left with seconds in the denominator. Similar reasoning follows the *kilometer* variable in the numerator. Doing the calculation yields:

$$(\text{km/hr})(\text{hr/sec})(\text{m/km}).$$

Putting in the numbers, we get:

(54 km/hr)[(1 hr)/(3600 sec)][(1000 m)/(1 km)] = 15 m/s.

b.) The relationship between the velocity of the *center of mass* of a rolling object (i.e., one that both translates and rotates) is:

$$v_{cm} = R\,\omega .$$

Minor side point: The temptation is to assume that the R term is simply the radius of the round object. Although the value for R is equal to the radius of the object in this case, R in this expression is really telling us how many meters there are on the arc of a one radian angle (remember, we are relating a linear measure v_{cm} to an angular measure ω). Using the relationship $v_{cm} = R\,\omega$, rearranging, then plugging in the numbers, we get:

$$\begin{aligned}\omega &= v_{cm}/R \\ &= (15 \text{ m/s})/(.3 \text{ m/rad}) \\ &= 50 \text{ rad/sec.}\end{aligned}$$

Note: Because we have been careful with our units, we get the correct units for the *angular velocity*.

As for *4 m/s* corresponding to *13.33 rad/sec*, we can use $v_{cm} = R\,\omega$ to write:

$$\begin{aligned}&4 \text{ m/s} = (.3 \text{ m})\ \omega \\ \Rightarrow\ &\omega = 13.33 \text{ rad/sec.}\end{aligned}$$

c.) If the *angular displacement* had been one rotation--2π radians--the distance traveled by a point on the wheel's edge would have been the arclength of an angular displacement $\Delta\theta$ equal to 2π radians, or $\Delta s = 2\pi R$ meters (remember, the relationship between arclength Δs, radius-to-the-point-in-question r, and angular displacement $\Delta\theta$ is $\Delta s = r\,\Delta\theta$). If the *angular displacement* had been two rotations--4π radians--the distance traveled would have been 4πR meters

If we lay the arclength Δs out flat, we will get the *total linear distance* the wheel traveled. We know that distance. If we additionally know the radius of the wheel, we can write:

$$\begin{aligned}&\Delta s = R\,\Delta\theta \\ \Rightarrow\quad &\Delta\theta = \Delta s/R \\ &\quad\ = (50 \text{ m})/(.3 \text{ m/rad}) \\ &\quad\ = 166.7 \text{ radians.}\end{aligned}$$

d.) EXTRA--wheel's angular acceleration?

$$(\omega_2)^2 = (\omega_1)^2 + 2\alpha(\theta_2 - \theta_1)$$
$$(13.33 \text{ rad/s})^2 = (50 \text{ rad/s})^2 + 2a(166.7 \text{ rad})$$
$$\Rightarrow \quad \alpha = -6.97 \text{ rad/s}^2.$$

e.) EXTRA--car's acceleration? We know the relationship between *angular acceleration* of a body and the magnitude of the *translational acceleration* of a point a distance *r* units from the axis of rotation. Using it we get:

$$\begin{aligned} a &= r\alpha \\ &= (.3 \text{ rad/m})(-6.97 \text{ rad/s}^2) \\ &= -2.09 \text{ m/s}^2. \end{aligned}$$

f.) EXTRA--time of motion? For elapsed time using $(\theta_2 - \theta_1) = \Delta\theta$, we get:

$$\Delta\theta = \omega_1 \Delta t + (1/2)\alpha(\Delta t)^2$$
$$(166.7 \text{ rad}) = (50 \text{ rad/s})t + .5(-6.97 \text{ rad/s}^2)t^2.$$

Using the quadratic formula, we get:

$$t = 5.27 \text{ seconds.}$$

g.) EXTRA--time using another way? For elapsed time using ω_2 and not $\Delta\theta$:

$$\alpha = (\omega_2 - \omega_1)/\Delta t$$
$$\Rightarrow \quad \Delta t = [(13.33 \text{ rad/s} - 50 \text{ rad/s})]/(-6.97 \text{ rad/s}^2)$$
$$= 5.26 \text{ sec} \qquad \text{(yes, } \textit{Parts f} \text{ and } g \text{ match).}$$

h.) EXTRA--average angular velocity?

$$\begin{aligned} \omega_{avg} &= (\omega_2 + \omega_1) / 2 \\ &= [(13.33 \text{ rad/s}) + (50 \text{ rad/s})]/2 \\ &= 31.67 \text{ rad/s.} \end{aligned}$$

i.) EXTRA--determine wheel's angular displacement $\Delta\theta$ after .5 seconds.

$$\begin{aligned}\Delta\theta &= \omega_1 \Delta t + (1/2)\alpha(\Delta t)^2 \\ &= (50 \text{ rad/s})(.5 \text{ s}) + .5(-6.97 \text{ rad/s}^2)(.5 \text{ s})^2 \\ &= 24.13 \text{ rad.}\end{aligned}$$

j.) EXTRA--how far does the car travel in first .5 seconds? *How far* translates as, "What was the arclength of the wheel's motion?"

$$\begin{aligned}\Delta s &= R\Delta\theta \\ &= (.3 \text{ m/rad})(24.13 \text{ rad}) \\ &= 7.24 \text{ m.}\end{aligned}$$

k.) EXTRA--wheel's final ω after .5 seconds <u>without</u> using *time* variable:

$$\begin{aligned}(\omega_3)^2 &= (\omega_1)^2 + 2\alpha\Delta\theta \\ &= (50 \text{ rad/s})^2 + 2(-6.97 \text{ rad/s}^2)(24.13 \text{ rad}) \\ &= 46.51 \text{ rad/s.}\end{aligned}$$

l.) EXTRA--angular displacement between .5 and .7 seconds?

$$\begin{aligned}\Delta\theta &= \omega_3 \Delta t + (1/2)\alpha(\Delta t)^2 \\ &= (46.51 \text{ rad/s})[(.7 \text{ s}) - (.5 \text{ s})] + .5(-6.97 \text{ rad/s}^2)(.2 \text{ s})^2 \\ &= 9.16 \text{ rad.}\end{aligned}$$

m.) EXTRA: From the information given in *Part b*, we know that the angular velocity of our wheel when moving at 4 m/s is 13.33 rad/sec. We can solve for $\Delta s = R\Delta\theta$ if we know $\Delta\theta$ during the motion (R is the radius of the wheel, or .3 m). We can use $\Delta\theta = \omega_1 \Delta t + (1/2)\alpha(\Delta t)^2$ if we know α. We start:

$$\begin{aligned}\alpha &= (\omega_4 - \omega_2) / \Delta t \\ &= [(20 \text{ rad/s}) - (13.33 \text{ rad/s})]/(3 \text{ s}) \\ &= 2.22 \text{ rad/s}^2.\end{aligned}$$

$$\begin{aligned}\Delta\theta &= \omega_2 \Delta t + (1/2)\alpha(\Delta t)^2 \\ &= (13.33 \text{ rad/s})(3 \text{ s}) + .5(2.22 \text{ rad/s}^2)(3 \text{ s})^2 \\ &= 49.98 \text{ radians.}\end{aligned}$$

$$\Delta s = R\Delta\theta$$
$$= (.3 \text{ m/rad})(49.98 \text{ rad})$$
$$= 14.99 \text{ meters.}$$

8.31) We are given the earth's mass at $m_e = 5.98x10^{24}$ *kg*, the earth's period T = *24 hours* ($8.64x10^4$ seconds--use dimensional analysis to get this if you don't believe me), and its radius at $r_e = 6.37x10^6$ *meters*.

a.) The earth rotates through 2π radians in 24 hours ($8.64x10^4$ seconds). Its angular displacement per unit time (i.e., its angular velocity) is, therefore:

$$\omega = \Delta\theta/\Delta t$$
$$= (2\pi \text{ rad})/(8.64x10^4 \text{ s})$$
$$= 7.27x10^{-5} \text{ rad/s.}$$

b.) The earth's equatorial velocity (magnitude) is equal to the linear distance a point on the equator travels per unit time. That is:

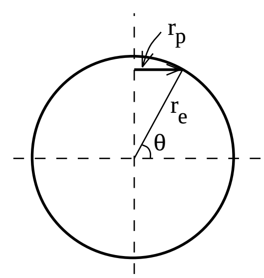
$$v_{eq} = (2\pi \text{ rad})(6.37x10^6 \text{ m/rad})/(8.64x10^4 \text{ s})$$
$$= 463.2 \text{ m/s} \qquad \text{(this is about 1000 mph).}$$

Note: According to theory, v_{equ} should equal $R\omega_e$. Putting the numbers in yields:

$$R\omega_e = (6.37x10^6 \text{ m})(7.27x10^{-5} \text{ rad/s})$$
$$= 463.1 \text{m/s.}$$

Given round-off error, this is close enough for government work.

c.) Looking at the sketch, the radius of the circle upon which that particle will be traveling will be:



$$r_p = r_e \cos 60^o$$
$$= (6.37x10^6 \text{ m})(.5).$$

It takes the same amount of time *T* for the particle at 60^o to travel through one rotation as it does for a particle on the equator, so:

$$\begin{aligned} v_p &= 2\pi r_p/T \\ &= (2\pi)(3.185\text{x}10^6 \text{ m})/(8.64\text{x}10^4 \text{ s}) \\ &= 231.6 \text{ m/s}. \end{aligned}$$

Does this make sense? Sure it does. As you approach the geographic north pole the travel velocity should go to zero.

d.) Using the table in Chapter 8, the *moment of inertia* of a solid sphere I_{ss} is:

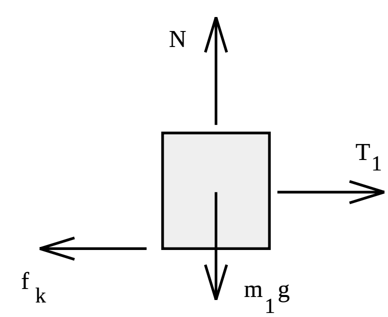
$$\begin{aligned} I_{ss} &= (2/5)MR^2 \\ &= .4(5.98\text{x}10^{24} \text{ kg})(6.37\text{x}10^6 \text{ m}))^2 \\ &= 9.7\text{x}10^{37} \text{ kg}\cdot\text{m}^2. \end{aligned}$$

<u>Bottom line:</u> It's going to take a mighty stiff cosmic breeze to change the earth's rotational motion!

8.32) The thing to remember whenever you have a massive pulley is that the tension on either side of the pulley will be different.

a.) This is a Newton's Second Law problem. F.b.d.'s are shown to the right and on the next page, and N.S.L. is presented below (notice that you end up needing FIVE independent equations to solve this problem):

<u>for mass m_1:</u>



$\underline{\sum F_y}$:

$$N - m_1g = m_1a_y = 0 \quad (\text{as } a_y = 0)$$
$$\Rightarrow \quad N = m_1g.$$

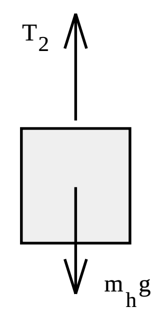
$\underline{\sum F_x:}$

$$-\mu_k N + T_1 = m_1 a$$
$$\Rightarrow \quad T_1 = m_1 a + \mu_k m_1 g.$$

Note: This presents a problem. We don't want the acceleration *a* of the mass m_1, we want the angular acceleration α of the pulley. We need a relationship between those two quantities. Noticing that the string's acceleration *a* is that of m_1 and the string's acceleration is also the acceleration *of a point on the pulley's circumference*, we can use the relationship $a = R\alpha$ for the job. Doing so yields:

$$T_1 = m_1 \quad a \quad + \mu_k m_1 g$$
$$T_1 = m_1(R\alpha) + \mu_k m_1 g.$$

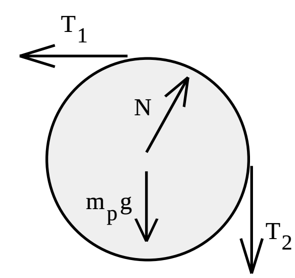
<u>for mass m_h:</u>



$\underline{\sum F_y:}$

$$T_2 - m_h g = -m_h a$$
$$\Rightarrow \quad T_2 = -m_h \quad a \quad + m_h g$$
$$\Rightarrow \quad T_2 = -m_h(R\alpha) + m_h g.$$

<u>for the pulley:</u>



$\underline{\sum \Gamma_{\text{pulley axis}}:}$

$$\Gamma_{T_1} - \Gamma_{T_2} = \quad - \quad I_p \quad \alpha$$
$$T_1 R - T_2 R = -[(1/2)M_p R^2]\,\alpha.$$

Note that the negative sign in front of the $I_p\alpha$ term denotes that the α term is a magnitude and the unembedded sign is designating a *clockwise angular acceleration*. Dividing out an *R* yields:

$$T_1 - T_2 = -[(1/2)M_p R]\,\alpha.$$

Substituting the expressions for T_1 and T_2 we get:

$$T_1 \qquad - \qquad T_2 \qquad = -[(1/2)M_p R]\,\alpha$$
$$[m_1(R\alpha) + \mu_k m_1 g\,] - [-m_h(R\alpha) + m_h g\,] = -[(1/2)M_p R]\,\alpha$$
$$\Rightarrow \quad \alpha = [-\mu_k m_1 g + m_h g] \,/\, [m_1 R + m_h R + .5M_p R].$$

Put in the numbers and you should come out with 29.32 rad/sec^2.

b.) The acceleration of a point on the edge of the pulley (this is also the acceleration of the string which, in turn, is the acceleration of the hanging mass), is $a = r\alpha$, where r in this case is the radius R of the pulley and α is provided from *Part a* above. Doing the work yields:

$$\begin{aligned} a &= R\alpha \\ &= (.1875 \text{ m/rad})(29.32 \text{ rad/sec}^2) \\ &= 5.5 \text{ m/s}^2. \end{aligned}$$

c.) The total work done by all the *tension* forces in the system equals zero. Noting that, the *conservation of energy* yields:

$$\begin{array}{llll} \Sigma KE_1 + \quad \Sigma U_1 + \Sigma W_{ext} = & \Sigma KE_2 & & + \Sigma U_2 \\ \quad 0 \quad + m_h gh \ + \ (\text{-}fh) & = [(1/2)m_1 v^2 + (1/2)m_h v^2 + (1/2)I_{cm}\omega^2] & & + \ 0. \end{array}$$

Noting that the velocity of a point on the pulley's circumference will equal v, and that $v = R\omega$, and the frictional force is $\mu_k N = \mu_k m_1 g$, we can write:

$$m_h gh = [(\mu_k m_1 gh) + (1/2)m_1(R\omega)^2 + (1/2)m_h(R\omega)^2 + (1/2)[I_{cm}\omega^2].$$

Rewriting this, eliminating the units (for space):

$$(1.2\,)(9.8)(1.5) = (.7)(.4)(9.8)(1.5) + .5(.4)(.1875)^2\omega^2 + .5(1.2)(.1875)^2\omega^2 + .5[1.4\text{x}10^{-3}]\omega^2$$

$$\Rightarrow \quad \omega = 21.66 \text{ rad/sec}.$$

d.) Noting that the string will have a velocity equal to that of both m_1 and m_h AND to a point on the circumference of the pulley, we can relate the angular velocity of the pulley and the string's velocity by $v = R\omega$. Remembering that the units for R in this usage are "meters/radian," we get:

$$\begin{aligned} v &= R\omega \\ &= (.1875 \text{ m/rad})(21.66 \text{ rad/sec}) \\ &= 4.06 \text{ m/s}. \end{aligned}$$

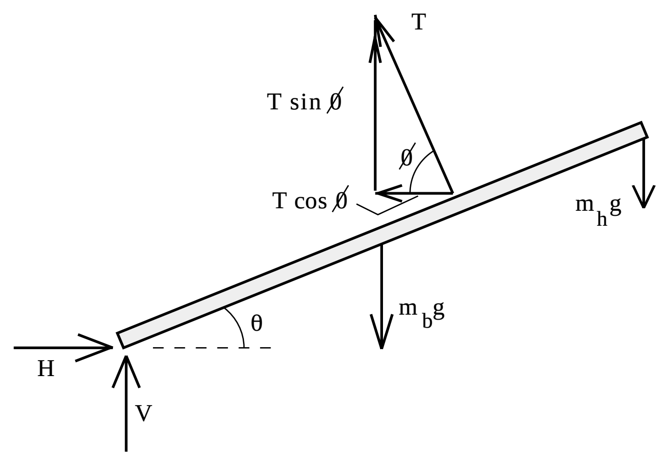
e.) This is a trick question. The acceleration is going to be the same no matter how far the hanging mass has fallen. The answer was derived in *Part b*.

f.) Angular momentum is defined rotationally as $L = I\omega$. With that we get:

$$\begin{aligned} L &= I\omega \\ &= [(1/2)m_pR^2]\,\omega \\ &= .5(.08\text{ kg})(.1875\text{ m})^2(21.66\text{ rad/sec}) \\ &= .03\text{ kg}\cdot\text{m}^2/\text{s}. \end{aligned}$$

8.33) An f.b.d. for the beam is shown below.

a.) This is a rigid body (i.e., equilibrium) problem. The easiest way to get the tension in the line is to sum the torques about the pin (that will eliminate the need to deal with *H* and *V* and will additionally give you an equation that has only one unknown--the *T* variable for which you are looking).



Note:

--Call the distance between the pin and the cable's connection to the beam $d = (2/3)L$, where L is the beam's length.

--The angle between T and d is 90^o which means the torque due to T about the pin will be $Td\sin 90^o = T(2/3)L$.

--The distance between the pin and the hanging mass's connection to the beam is L.

--The component of L perpendicular to the *line of the hanging mass* (i.e., *r-perpendicular*) will be $L\cos 30^o$ (this is like determining the shortest distance between the pin and the *line of the force*). That means the torque provided by the hanging mass will be $(m_hg)L\cos 30^o$. As this torque will try to make the beam rotate clockwise, it is a *negative* torque.

--The *mass of the beam* can be assumed to be located at the beam's *center of mass* at *L/2*.

Putting all this together, N.S.L. yields:

$\underline{\sum \Gamma_{pin}:}$

$$\Gamma_T \quad + \quad \Gamma_{m_h} \quad + \quad \Gamma_{m_b} \quad = 0 \quad (\text{as } \alpha = 0)$$

$$T[(2/3)L] - (m_h g)L\cos 30^o - (m_b g)(L/2)\cos 30^o = 0.$$

Canceling the *L's* and solving for *T*, we get:

$$\begin{aligned} T &= [(m_h g)\cos 30^o + (m_b g)(.5)\cos 30^o]/(2/3) \\ &= [(3\text{ kg})(9.8\text{ m/s}^2)(.87) + (7\text{ kg})(9.8\text{ m/s}^2)(.5)(.87)]/[.67] \\ &= 82.7\text{ nts.} \end{aligned}$$

Knowing *T*, we can sum the forces in the *x* and *y directions* to determine *H* and *V*. Doing so yields:

$\underline{\sum F_x:}$

$$\begin{aligned} -T\cos 60^o + H &= ma_x \\ &= 0 \quad \text{as } a_x = 0. \end{aligned}$$

$$\begin{aligned} \Rightarrow \quad H &= T\cos 60^o \\ &= (82.7\text{ nt})(.5) \\ &= 41.35\text{ nts.} \end{aligned}$$

$\underline{\sum F_y:}$

$$\begin{aligned} T\sin 60^o + V - m_h g - m_b g &= ma_y \\ &= 0 \quad (\text{as } a_y = 0) \end{aligned}$$

$$\begin{aligned} \Rightarrow \quad V &= -T\sin 60^o + m_h g + m_b g \\ &= -(82.7\text{ nt})(.87) + (3\text{ kg})(9.8\text{ m/s}^2) + (7\text{ kg})(9.8\text{ m/s}^2) \\ &= 26.05\text{ nts.} \end{aligned}$$

Note: If there had been a negative sign in front of the 26.05 newtons, it would have meant that we had assumed the wrong direction for the force *V*. The *magnitude* would nevertheless have been correct.

b-i.) The *moment of inertia* about an axis other than one through the *center of mass* but parallel to an axis of known *moment of inertia* that is through the *center of mass* is determined using the *Parallel Axis Theorem*. The *moment of inertia* about the beam's pin is:

$$\begin{aligned} I_p = \quad & I_{cm} \quad + \quad mh^2 \\ &= (1/12)m_bL^2 + m_b(L/2)^2 \\ &= (1/3)m_bL^2 \\ &= (1/3)(7\text{ kg})(1.7\text{ m})^2 \\ &= 6.74\text{ kg}\cdot\text{m}^2. \end{aligned}$$

b-ii.) The *moment of inertia* of a point mass (i.e., the hanging mass) r units from a reference axis (note that $r = L$ in this problem) is:

$$\begin{aligned} I_{hm} &= mr^2 \\ &= m_hL^2 \\ &= (3\text{ kg})(1.7\text{ m})^2 \\ &= 8.67\text{ kg}\cdot\text{m}^2. \end{aligned}$$

b-iii.) The total *moment of inertia* about the pin is the beam's *moment of inertia* about the pin added to the hanging mass's *moment of inertia* about the pin, or:

$$\begin{aligned} I_{tot,pin} &= \quad I_p \quad + \quad I_{hm} \\ &= (6.74\text{ kg}\cdot\text{m}^2) + (8.67\text{ kg}\cdot\text{m}^2) \\ &= 15.4\text{ kg}\cdot\text{m}^2. \end{aligned}$$

c.) Angular acceleration--think N.S.L. Summing up the torques *about the pin* (the beam is executing a pure rotation about the pin--no reason to sum torques about any other point) and putting that equal to the *moment of inertia* ABOUT THE PIN times the *angular acceleration* about the pin yields:

$\underline{\sum \Gamma_{pin}:}$

$$\Gamma_T \quad + \quad \Gamma_{m_{hm}} \quad + \quad \Gamma_{m_b} \quad = I_{tot,pin}\,\alpha$$

$$T[(2/3)L] - (m_hg)L\cos 30^o - (m_bg)(L/2)\cos 30^o = I_{tot,pin}\,\alpha.$$

The line has been cut (which means $T = 0$) and we know the total *moment of inertia* about the pin from above. Using this we get:

$$\Gamma_T \quad + \quad \Gamma_{m_h} \quad + \quad \Gamma_{m_b} \quad = \quad I_{tot,pin}\,\alpha$$
$$0 \quad - (m_h g)L\cos 30^o - (m_b g)(L/2)\cos 30^o = -(15.4\ kg\cdot m^2)\,\alpha.$$

Solving for α we get:

$$\begin{aligned}\alpha &= [(m_h g)L\cos 30^o + (m_b g)(L/2)\cos 30^o)]/(15.4\ kg\cdot m^2)\\ &= [(3\ kg)(9.8\ m/s^2)(1.7\ m)(.87) + (7\ kg)(9.8\ m/s^2)(.85\ m)(.87)]/(15.4\ kg\cdot m^2)\\ &= 6.11\ rad/sec^2.\end{aligned}$$

d.) Knowing the *angular acceleration* of the beam just after it lets loose allows us to determine the *translational acceleration* of any point on the beam using $a = r\alpha$, where r is the distance between the pin and the point-in-question. In this case, that distance is $L/2$. Using this information, we get:

$$\begin{aligned}a &= r\alpha\\ &= [(1.7\ m)/2]\ (6.11\ rad/sec^2)\\ &= 5.19\ m/s^2.\end{aligned}$$

e.) This is a *conservation of energy* problem. The beam has *potential energy* wrapped up in the fact that its *center of mass* will have fallen a vertical distance equal to $(L/2)\sin 30^o$ during the freefall. That means that its initial *potential energy* will be $m_b g(L/2)\sin 30^o$ while its final U will be zero. The hanging mass also falls a vertical distance equal to $L\sin 30^o$.

The *potential energy* changes of both bodies (i.e., the beam and hanging mass) must be taken into account if we are going to use *conservation of energy*--the approach of choice whenever we are looking for a velocity-type variable.

Assuming we approach the beam as though it were executing a pure rotation about its pin, we can relate the change in the potential energy of the system before the snip to the *rotational kinetic energy* of the beam about the pin plus the *translational kinetic energy* of the hanging mass just as the beam becomes horizontal. Using that approach:

$$\sum KE_1 + \quad \sum U_1 \quad + \sum W_{ext} = \quad \sum KE_2 \quad + \sum U_2$$
$$0 \quad + \left[m_b g((L/2)\sin 30^o) + m_h g(L\sin 30^o)\right] + 0 \quad = \left[(1/2)m_h v^2 + (1/2)I_p\omega^2\right] + \quad 0.$$

Noting that the velocity of a point on the beam will equal $v = R\omega$, we can write the velocity of the hanging mass as $v_h = (L)\omega$:

$$m_b g((L/2)\sin 30^o + m_h g(L\sin 30^o) = (1/2)m_h \ \ v_h{}^2 \ + (1/2) \qquad I_p \qquad \omega^2$$
$$m_b g((L/2)\sin 30^o + m_h g(L\sin 30^o) = (1/2)m_h[L\omega]^2 + (1/2)[(1/3)m_b L^2]\,\omega^2.$$

Putting in the numbers while eliminating the units (for space), we get:

$$(7)(9.8)(.85)(.5) + (3)(9.8)(1.7)(.5) = .5(3)(1.7^2)w^2 + .5(.33)(7)(1.7)^2\omega^2$$
$$\Rightarrow \quad \omega = 2.62 \text{ rad/sec.}$$

f.) The *translational velocity* of the *center of mass* will be:

$$\begin{aligned} v &= (L/2)\ \omega \\ &= [(1.7 \text{ m})/2](2.62 \text{ rad/sec}) \\ &= 2.23 \text{ m/s.} \end{aligned}$$

g.) *Angular momentum* is defined rotationally as $L = I\omega$, where I is the TOTAL *moment of inertia* for the beam and hanging mass about the pin, and the fact that the length of the beam has been defined as L turns out to be a really, really bad choice of variables (it's the same as *angular momentum*). Remembering that $I_{tot,pin} = 15.4\ kg{\cdot}m^2/s$, we get:

$$\begin{aligned} L &= I\omega \\ &= [15.4](2.62 \text{ rad/sec}) \\ &= 40.35 \text{ kg}{\cdot}\text{m}^2/\text{s.} \end{aligned}$$

h.) *Angular momentum* is conserved *if and only if* all the torques acting on the system are internal (i.e., are the consequence of the interaction of the various parts of the system). Gravity is an external force which means that any torque it produces on the beam will be an *external* torque. In short, *angular momentum* should not be conserved.

(This should be obvious given the fact that the beam is accelerating angularly).

8.34) The merry-go-round's mass is $m_m = 225\ kg$ while each child's mass is $m_c = 35\ kg$. The radius of the merry-go-round is $R = 2.5$ *meters* and its *angular velocity* is $\omega_1 = .8$ *radians/second* when the kids climb on.

a.) An *angular velocity* of $\omega_1 = .8$ *rad/sec* produces a *translational velocity* of $r\omega = (2.5\ m)(.8\ rad/sec) = 2\ m/s$ at the edge of the merry-go-round (i.e., where the children are when they first jump on). Also, the *moment of inertia* of one child when he or she first jumps onto the merry-go-round is $I_{c,1} = m_c R^2 = (35\ kg)(2.5\ m)^2 = 218.75 kg{\cdot}m^2$ while the *moment of inertia* of the merry-go-round itself is assumed to be that of a disk and is equal to $I_m = (1/2)m_m R^2 = .5(225\ kg)(2.5\ m)^2 = 703\ kg{\cdot}m^2$. Using *conservation of energy,* we can write:

$$\begin{array}{ccccccccc} \sum KE_1 & + & \sum U_1 & + & \sum W_{ext} & = & \sum KE_2 & + & \sum U_2 \\ 0 & + & 0 & + & 3[(RF)\Delta\theta] & = & \left[3(1/2)I_c\omega_2^{\ 2} + (1/2)I_m\omega_2^{\ 2}\right] & + & 0. \end{array}$$

Note: $\Gamma_{one\ kid\ on\ mgr} = RF \sin 90^o$. The counterpart to $\mathbf{F}{\cdot}\mathbf{d}$ for rotational motion is $\mathbf{\Gamma}\cdot\Delta\boldsymbol{\theta}$, so the three kids do $3[(RF)\Delta\theta]$ worth of extra work on the m.g.r.

Solving, we get:

$$3(2.5\ m)(15\ nt)\Delta\theta = \left[3(.5)(218.75\ kg{\cdot}m^2)(.8\ rad/s)^2 + .5(703\ kg{\cdot}m^2)(.8\ r/s)^2\right]$$
$$\Rightarrow \quad \Delta\theta = 3.87 \text{ radians.}$$

Note that the relationship between angular displacement and linear displacement is:

$$\begin{aligned} \Delta s &= R\Delta\theta \\ &= (3.87\ rad)(2.5\ m/rad) \\ &= 9.68 \text{ meters.} \end{aligned}$$

b.) Once the kids have climbed aboard, the torques acting on the system are all internal. That is, they are due to the interaction of the system's parts (the kids act on the merry-go-round while the merry-go-round acts on the kids). That means we can use the *conservation of* *angular momentum:*

$$L_{1,tot} = L_{2,tot}$$
$$L_{m,1} \quad + \quad L_{c,1} \quad = \quad L_{m,2} \quad + \quad L_{c,2}$$

$$I_m \omega_1 + (I_{c,1} \omega_1) = I_m \omega_2 + [I_{c,2} \omega_2]$$

$$(703\ kg\cdot m^2)(.8\ r/s) + (656\ kg\cdot m^2)(.8\ r/s) = (703\ kg\cdot m^2)(\omega_2) + [3(35\ kg)(1\ m)^2\ \omega_2]$$

$$\Rightarrow \quad \omega_2 = 1.35 \text{ rad/sec and}$$

$$\begin{aligned} \Rightarrow \quad v_2 &= r\omega \\ &= (1 \text{ m/rad})(1.346 \text{ rad/sec}) \\ &= 1.35 \text{ m/s.} \end{aligned}$$

c.) As there are no external torques, the *angular momentum* should be conserved.

Also, the forces acting on the system are *internal* so the *total momentum* of the system will be conserved as the children move toward the merry-go-round's center. It should be noted, though, that this isn't a very useful fact as far as problem-solving goes. The addition of the momentum components of the children's motion at the beginning *and at any time thereafter* will be zero.

d.) The forces being exerted on the system are not conservative (the kids burn chemical energy as they move on the merry-go-round), so energy is not conserved.

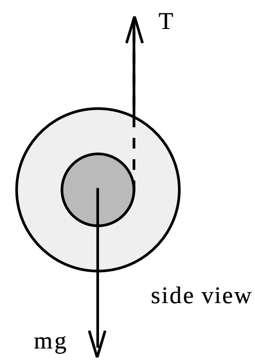
e.) We have already deduced that energy should not be conserved. Let's see if we were right by calculating the energy in the system just after the kids jumped onto the merry-go-round and the energy once they reached the *r = 1 meter* point:

$$\begin{aligned} KE_1 &= KE_{c,1} + KE_{m,1} \\ &= (1/2)m_{c,1}v_1^2 + (1/2)I_m\omega_1^2 \\ &= .5[3(35\ kg)](2\ m/s)^2 + .5(703\ kg\cdot m^2)(.8\ r/s)^2 \\ &= 435 \text{ joules.} \end{aligned}$$

$$\begin{aligned} KE_2 &= KE_{c,2} + KE_{m,2} \\ &= (1/2)m_{c,2}v_2^2 + (1/2)I_m\omega_2^2 \\ &= .5[3(35\ kg)](1.35\ m/s)^2 + .5(703\ kg\cdot m^2)(1.35\ r/s)^2 \\ &= 736.3 \text{ joules.} \end{aligned}$$

Energy is obviously not conserved.

8.35) Among other reasons, this problem was designed to allow you to see that either the *pure rotation* or the *rotation about the center of mass plus translation of the center of mass* approaches will work when dealing with either a Newton's Second Law or *conservation of energy* problem. A f.b.d. for the problem is shown to the right.



a.) Analyzing the wheel problem from a *rotation about the center of mass plus translation of the center of mass* approach:

This is a N.S.L. problem. We will begin by summing the torques about the center of mass:

$$\underline{\sum \Gamma_{cm}:}$$

$$Tr_a = I_{cm}\alpha \qquad \text{(Equation A).}$$

We have two unknowns here; we need another equation. Consider the sum of the forces in the vertical:

$$\underline{\sum F_v:}$$

$$\begin{aligned} T - mg &= -ma \\ &= -m(r_a\alpha) \end{aligned}$$

$$\Rightarrow \quad T = mg - mr_a\alpha \qquad \text{(Equation B).}$$

Putting *Equations A* and *B* together:

$$\begin{aligned} &Tr_a = I_{cm}\alpha \\ \Rightarrow \quad &(mg - mr_a\alpha)r_a = I_{cm}\alpha \\ \Rightarrow \quad \alpha &= (mgr_a)/(I_{cm} + mr_a^2) \\ &= [(.6 \text{ kg})(9.8 \text{ m/s}^2)(.015 \text{ m})]/[(1.2\text{x}10^{-4} \text{ kg}\cdot\text{m}^2) + (.6 \text{ kg})(.015)^2] \\ &= 346 \text{ rad/sec}^2. \end{aligned}$$

b.) We have to use the Parallel Axis Theorem to determine the *moment of inertia* about the new axis. Doing so yields:

$$\begin{aligned} I_a &= I_{cm} + m h^2 \\ &= (1.2\text{x}10^{-4} \text{ kg}\cdot\text{m}^2) + (.6 \text{ kg})(.015 \text{ m})^2 \\ &= 2.55\text{x}10^{-4} \text{ kg}\cdot\text{m}^2. \end{aligned}$$

c.) Analyzing the wheel problem from a *pure rotation* approach:

This is a N.S.L. problem. We will begin by summing the torques about the axis .015 meters from the *center of mass* (call this *Point P*).

$\underline{\sum \Gamma_a:}$

$$(mg)r_a = I_a \alpha.$$

We determined I_a in *Part b*, so all we have to do is solve for α:

$$\begin{aligned}\alpha &= (mgr_a)/I_a \\ &= [(.6 \text{ kg})(9.8 \text{ m/s}^2)(.015 \text{ m})]/(2.55\text{x}10^{-4} \text{ kg}\cdot\text{m}^2) \\ &= 346 \text{ rad/sec}^2.\end{aligned}$$

Great jumping Huzzahs! *Parts a* and *c* match. Both approaches work.

d.) This is a *conservation of energy* problem. Major note: *Work* requires that a force act *through a distance.* Tension acts at the one point on the disk that *isn't moving*--it acts at the instantaneously stationary point about which the disk rotates. As such, tension acts through *no distance* and does *no work.*

--from the *pure rotation* approach:

$$\begin{array}{ccccccccc} \sum KE_1 & + & \sum U_1 & + & \sum W_{ext} & = & \sum KE_2 & + & \sum U_2 \\ 0 & + & mgd & + & 0 & = & (1/2)I_a\omega^2 & + & 0 \\ 0 & + & (.6 \text{ kg})(9.8 \text{ m/s}^2)(.18 \text{ m}) & + & 0 & = & .5(2.55\text{x}10^{-4} \text{ kg}\cdot\text{m}^2)\omega^2 & + & 0 \end{array}$$

$$\Rightarrow \quad \omega = 91.1 \text{ rad/sec.}$$

--from the *rotation about the center of mass plus translation of the center of mass* approach, noting that $v_{cm} = r_a\omega$:

$$\begin{array}{ccccccccc} \sum KE_1 & + & \sum U_1 & + & \sum W_{ext} & = & \sum KE_2 & + & \sum U_2 \\ 0 & + & mgd & + & 0 & = & [\ (1/2)I_{cm}\omega^2 + (1/2)mv_{cm}^2\ ] & + & 0 \\ 0 & + & (.6 \text{ kg})(9.8 \text{ m/s}^2)(.18 \text{ m}) & + & 0 & = & [.5(1.2\text{x}10^{-4} \text{ kg}\cdot\text{m}^2)\omega^2 + .5(.6 \text{ kg})[(.015 \text{ m})\omega]^2] & + & 0 \end{array}$$

$$\Rightarrow \quad \omega = 91.1 \text{ rad/sec.}$$

Again, we get the same answer no matter which approach we use.

e.) This is trivial, given the fact that you know the *angular velocity* calculated in *Part d*.

$$\begin{aligned} v_{cm} &= r_a \omega \\ &= (.015 \text{ m/rad})(91.1 \text{ rad/sec}) \\ &= 1.37 \text{ m/s.} \end{aligned}$$

8.36)

a.) This is a collision problem. *Energy* is not conserved through the collision because non-conservative forces act during the collision and because the collision was not close enough to being lossless to be approximated as *elastic*. *Momentum* is not conserved because the pin applies an external force to the rod. *Angular momentum* is conserved as there are no external torques (the external force at the pin applies no torque to the system because the pin force is applied at the axis of rotation).

As the rod is massless, we can treat all of the masses in the system as point masses. We know that there are two ways to calculate angular momentum (i.e., by using the magnitude of ***rx(mv)*** or by using $I\omega$), and we know that the *moment of inertia* of a point mass is mr^2 (this means the total moment of inertia of the system is $m_1(d/2)^2 + 2m_2(d/2)^2 = (m_1 + 2m_2)(d/2)^2$). With all of this, we can write:

$$\begin{aligned} \Sigma L_o &= \Sigma L_f \\ L_{wad,o} + 2L_{mass,o} &= I_{tot}\ \omega_1 \\ m_1 v_o (d/2) \sin 90^o + 0 &= [(m_1 + 2m_2)(d/2)^2]\ \omega_1 . \end{aligned}$$

Plugging in the numbers, we get:

$$(.9 \text{ kg})(2.8 \text{ m/s})(1.2/2 \text{ m}) \sin 90^o = [(.9 \text{ kg} + 2(2 \text{ kg}))(1.2/2 \text{ m})^2]\,\omega_1$$

$$\Rightarrow \quad \omega_1 = .857 \text{ rad/sec.}$$

$$\begin{aligned} \Rightarrow \quad v_{at\ ends} &= (d/2)\,\omega_1 \\ &= (1.2/2 \text{ m})(.857 \text{ rad/sec}) \\ &= .514 \text{ m/s.} \end{aligned}$$

b.) As there is essentially no *potential energy* change through the collision, the *total energy change* will be wrapped up in the *kinetic energy* difference. The initial *kinetic energy* is all translational, being associated with the wad. The final *kinetic energy* can be treated as purely rotational. Doing so yields an energy difference of:

$$\Delta KE = KE_{after} - KE_{bef}$$
$$= (1/2)\, I_{tot}\, \omega_1^{\ 2} - (1/2)(m_1)v_o^{\ 2}$$
$$= .5\,[(m_1 + 2m_2)(d/2)^2]\, \omega_1^{\ 2} - .5(m_1)v_o^{\ 2}.$$

Putting in the numbers we get:

$$\Delta KE = .5[(.9\text{ kg} + 2(2\text{ kg}))(1.2/2\text{ m})^2](.857\text{ rad/sec})^2 - .5(.9\text{ kg})(2.8\text{ m/s})^2$$
$$= -2.884\text{ joules} \quad \text{(energy is lost).}$$

c.) The net angular displacement of the system can be determined using *conservation of energy* for the motion *after the collision*. In fact, the symmetry of the problem makes the calculation particularly easy as the *potential energy* picked up by the *right mass* in its transit is equal to the potential energy lost by the *left mass*. As such, the only *potential energy* gain will be associated with the position-change of the wad. That means:

$$\begin{array}{ccccccccc} \Sigma KE_1 & + & \Sigma U_1 & + & \Sigma W_{ext} & = & \Sigma KE_2 & + & \Sigma U_2 \\ (1/2)I_{tot}\omega_1^{\ 2} & + & 0 & + & 0 & = & 0 & + & m_1 gh. \end{array}$$

where h is the vertical distance above the wad's initial position. Solving for h yields:

$$h = (1/2)I_{tot}\,\omega_1^{\ 2}/(m_1 g)$$
$$= (1/2)[(.9\text{ kg} + 2(2\text{ kg}))(1.2/2\text{ m})^2](.857\text{ rad/sec})^2 / [(.9\text{ kg})(9.8\text{ m/s}^2]$$
$$= .073\text{ meters.}$$

With h, we can form a right triangle whose hypotenuse is $d/2 = .6$ *meters* and whose *opposite side* is $h = .073$ *meters* (see the triangle formed in Figure IV on the previous page). Taking the *inverse sine* yields an angle of .122 radians. That means the rod's net angular displacement will be π radians plus .122 radians, or 3.26 radians.

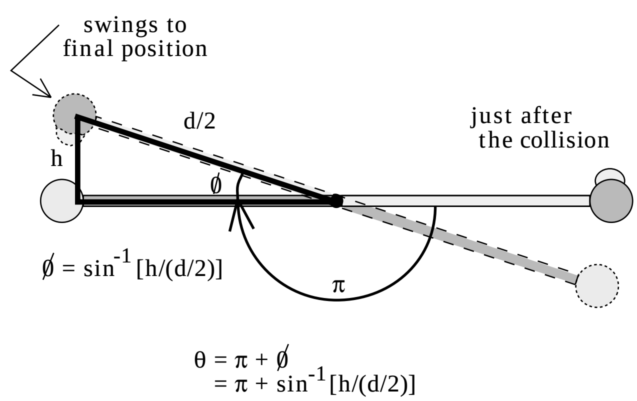


FIGURE IV

Note: As we know the energy lost from *Part b*, we could have used the *modified conservation of energy* equation *through the collision*. Doing so would have yielded:

$$\begin{aligned}
\Sigma KE_o \quad + \quad \Sigma U_o + \Sigma W_{ext} &= \Sigma KE_1 + \Sigma U_1 \\
(1/2)m_1 v_o^2 \quad + \quad 0 \quad + (-2.884\ j) &= \quad 0 \quad + m_1 gh \\
\Rightarrow \quad (1/2)(.9\ kg)(2.8\ m/s)^2 + \quad (-2.884\ j) &= (.9\ kg)(9.8\ m/s^2)h \\
\Rightarrow \quad h &= .073\ \text{meters.}
\end{aligned}$$

8.37) This obviously has within it a collision problem. Through the collision, *energy* is not conserved because the forces involved in the collision are undoubtedly non-conservative and because the problem's author has not deemed it necessary to imbue the collision with the magical label of *elastic*. *Momentum* is not conserved due to the fact that an external force is provided by the pin. *Angular momentum* is conserved because the external force at the pin produces no torque on the system.

If we can determine the amount of energy there is in the system just after the collision, we can use conservation of energy to determine the rise of the stick's center of mass after the collision and, from that, the final angle of the stick. As the *conservation of angular momentum* will allow us to determine the angular velocity of the system just after the collision and, with that, the energy in the system just after the collision, we will begin there.

a.) Assume the before-collision velocity of the block (i.e., after free falling to the bottom of the incline) is v_1 and the after-collision angular velocity of the stick is ω_2. Also, assume the block stays stationary just after the collision. With all that, the angular momentum about the pin is:

$$\begin{aligned}
L_{before} \quad &= \quad L_{after} \\
L_{block,bef} + L_{rod,bef} &= L_{rod,aft} + L_{block,aft} \\
mv_1 d \sin 90^o + \quad 0 &= I_{p,rod}\ \omega_2 + \quad 0 \\
&= [(1/3)(5m)d^2]\ \omega_2 \\
\Rightarrow \quad \omega_2 &= (3/5)v_1/d.
\end{aligned}$$

The problem here? We don't know what v_1 is. To determine that quantity, use *conservation of energy* during the block's slide down the frictionless incline. Doing so yields:

$$\Sigma KE_o + \Sigma U_o + \Sigma W_{ext} = \Sigma KE_1 + \Sigma U_1$$

$$0 \quad + mg(.4d) \; + \quad 0 \quad = (1/2)mv_1^{\,2} + \quad 0$$

$$\Rightarrow \quad v_1 = [2g(.4d)]^{1/2}$$

$$=2.8(d)^{1/2}.$$

With v_1, the angular velocity of the stick just after the collision becomes:

$$\begin{aligned} \omega_2 &= (3/5)v_1/d \\ &= (3/5)[2.8(d)^{1/2}/d] \\ &= 1.68/(d)^{1/2}. \end{aligned}$$

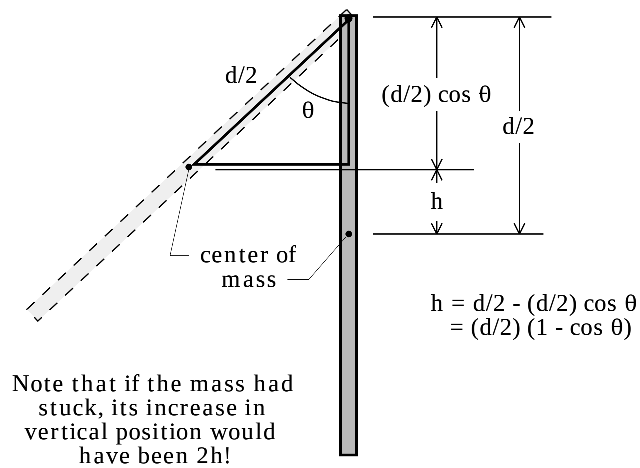


FIGURE V

We are now in a position to use *conservation of energy* from the time just after the collision to the time when the stick gets to the top of its motion (see Figure V). Defining the position of both the center of mass of the stick and the position of the block *just after the collision* to be the *potential energy equals zero* level for each object, we can write:

$$\begin{array}{ccccccccc} \sum KE_2 & + & \sum U_2 & + \sum W_{ext} = & \sum KE_3 & + & \sum U_3 \\ [KE_{2,bl} + KE_{2,st}] & + & [U_{2,bl} + U_{2,st}] & + \sum W_{ext} = & [KE_{3,bl} + KE_{3,st}] & + & [U_{3,bl} + U_{3,st}] \\ [\; 0 \; + (1/2)I_p\omega_2^{\,2}] & + & [\; 0 \; + \; 0 \;] & + \; 0 \; = & [\; 0 \; + \; 0 \;] & + & [\; 0 \; + (5m)gh]. \end{array}$$

Solving:

$$\Rightarrow \quad .5 \quad I_{p,st} \quad \omega_2^{\,2} \; = \; (5m)\, g \quad h$$

$$.5\left[(1/3)(5m)d^2\right] \left[1.68/(d)^{1/2}\right]^2 = (5m)\, g \left[(d/2)(1 - \cos\theta)\right]$$

$$\Rightarrow \quad \theta = .44 \text{ radians or } 25.3^o.$$

b.) Assume the block adheres to the stick after the collision. Assume also that the before-collision velocity of the block (i.e., after free falling to the bottom of the incline) is v_1, the block's *moment of inertia* about the pin is md^2, and the after-collision angular velocity of both the block and stick is ω_2. With all this, we can write:

$$L_{before} = L_{after}$$
$$L_{block,bef} + L_{rod,bef} = L_{rod,aft} + L_{block,aft}$$
$$mv_1 d \sin 90^o + 0 = I_{pin,rod}\,\omega_2 + I_{p,bl}\,\omega_2$$
$$\Rightarrow \quad mv_1 d = [(1/3)(5m)d^2 + md^2]\,\omega_2.$$

Canceling *d's* and *m's* and simplifying, we get:

$$\omega_2 = .375v_1/d.$$

What is v_1? As derived in *Part a*:

$$v_1 = 2.8d^{1/2}.$$

With v_1, the angular velocity of the stick just after the collision becomes:

$$\begin{aligned}\omega_2 &= .375\quad v_1 \quad /d\\ &= .375[2.8(d)^{1/2}]/d\\ &= 1.05/d^{1/2}.\end{aligned}$$

We are now in a position to use *conservation of energy* from the time just after the collision to the time when the stick gets to the top of its motion (Figure V still captures the spirit of this calculation). Defining the position of the center of mass of the meter stick and the position of the block *just after the collision* to be the *potential energy equals zero* levels for each object, remembering that the moment of inertia of the block is md^2, and noting that if the stick's center of mass rises a distance h, the block rises a distance $2h$ (see Figure V), we can write:

$$\sum KE_2 + \sum U_2 + \sum W_{ext} = \sum KE_3 + \sum U_3$$
$$[KE_{2,bl} + KE_{2,st}] + 0 + 0 = [KE_{3,bl} + KE_{3,st}] + [U_{3,bl} + U_{3,st}]$$
$$[.5(md^2)\,\omega_2^{\,2} + .5I_p\,\omega_2^{\,2}] + 0 + 0 = [\ 0 + 0\ ] + [mg(2h) + (5m)gh].$$

Noting that the stick's *moment of inertia* about the pin is $I_p = (1/3)(5m)d^2 = (5/3)md^2$, we can simplify to get:

$$.5md^2\,\omega_2^{\,2} + .5[(5/3)md^2]\,\omega_2^{\,2} = mg[2(d/2)(1 - \cos\theta)] + (5m)g[(d/2)(1 - \cos\theta)]$$
$$\Rightarrow \quad [.5md^2 + (5/6)md^2]\,\omega_2^{\,2} = [2mg + (5m)g]\ [(d/2)(1 - \cos\ \theta)]$$

$$\Rightarrow \quad 1.33md^2\omega_2{}^2 = (7/2)mgd(1 - \cos\theta)].$$

Using $\omega_2 = 1.05/d^{1/2}$, we find:

$$1.33md^2[1.05/d^{1/2}]^2 = (7/2)mgd - (7/2)mgd\cos\theta.$$

Canceling *m's* and the *d's*, we get:

$$1.33[1.05^2] = (7/2)g - (7/2)g\cos\theta$$
$$\Rightarrow \quad \theta = .293 \text{ radians or } 16.8^{o}.$$